№ 4 (2), 2020 ISSN: 2682 – 9517 (print) ISSN: 2683 – 0930 (online)
INSTITUTE OF INFORMATICS AND INNOVATIVE TECHNOLOGIES

OPTIMAL DECISION MAKING BY DIFFERENT GENERALIZED DESIRABILITY FUNCTIONS

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ОПТИМАЛНО ВЗЕМАНЕ НА РЕШЕНИЯ ПРИ РАЗЛИЧНИ ОБОБЩЕНИ ФУНКЦИИ НА ЖЕЛАТЕЛНОСТ

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***Abstract* — New kind of strategies for optimal decision making are proposed and investigated. The strategies are based on the generalized functions of desirability. The variety of creating the generalized function of desirability includes various transformations of the several objective parameters in linear, exponential and multi-exponential type.**

The sensitivity of the new generalized functions of usefulness is tested by a practical problem to choose an optimal offer of a trade product. The results show that the best practical property has the multiplicative type and the new strategies are more sensitive to the control variables changes.

Keywords— decision making, Pareto-optimal solution, utility function, function of rationality, additive generalized function, multiplicative type function, harmonic type function, arithmetic mean, geometric mean, harmonic mean.

INTRODUCTION

Optimal decision-making is necessary when we have many possible alternative and competing solutions, and one of them must be chosen. For example, in the optimization of technological processes with a large number of target indicators, or when assessing the quality of an industrial product with many indicators of product quality. In this cases it is necessary to to find a technological process control, which will result in many possible solutions, which, in addition to meeting the technological requirements of the product and the requirements of the relevant norms and standards, but must also meet the users requirements of this product. The chosen solution will be a compromise solution, also called Pareto - optimal solution [1, 3]. This also applies to the cases when we have a certain set number of possible alternative solutions and it is reasonable to choose one optimal solution, or there are many offers and the optimal offer must be chosen.

Let m target indicators are set,

$\mathbf{y} \in (y_1(\mathbf{x}), y_2(\mathbf{x}), \ldots, y_m(\mathbf{x}))$,

which characterize the functioning of a given object or system, under different control parameters

$\mathbf{x} \in (x_1, x_2, \ldots, x_n)$ are set.

For the control parameters the admissible limits of their change are set $x_{i,min}$ and $x_{i,max}$. For each of the targets $y_j(\mathbf{x})$, $j = 1, 2, \ldots, m$, it is required to have an optimal value (maximum or minimum). The task is to find an optimal control $\mathbf{x}^* \in (x_1^*, x_2^*, \ldots, x_n^*)$, where each of the target parameters will obtain the desired optimal value $y_j^{opt}(\mathbf{x}^*)$.

STRATEGIES FOR GENERALIZED TARGET FUNCTIONS CREATION

The justification of the chosen optimal decision strongly depends on the chosen decision-making strategy. For this purpose, usually the set of criteria is transformed into one generalized criterion. Different strategies are used [1,2,3,4,5]. Depending on the chosen strategy for creating of generalized target function, which is maximized or minimized, there is an optimal management/control and a compromise optimal solution that will best meet the requirements of all target indicators. The most used generalized

functions are created to maximize the desired, useful, satisfactory, rational solutions. In other cases, generalized functions are used, which should minimize the losses from the recommended decisions or minimize the regret from the decision taken.

The newest and most promising strategies in recent years are the combined strategies, which seek rationality in decision-making [8].

In rational optimal decisions making, the chosen form of transformation has the greatest role, in which the natural values of the set target indicators are normalized so that all physical, economic, social and other indicators are brought into the same dimensionless scale, for example [0 +1]. The methods and strategies for this transformation have a continuous development in the time.

In 1832, Benjamin Gompertz published a series of sigmoid exponential growth functions [6], known as Gompertz curves, which represent mathematical models of human development throughout life. One of the Gompertz curves is shown as an example in Fig. 1. Later, Charles Winsor in 1932 [7] described the first attempts to use Gompertz functions to describe growth processes.

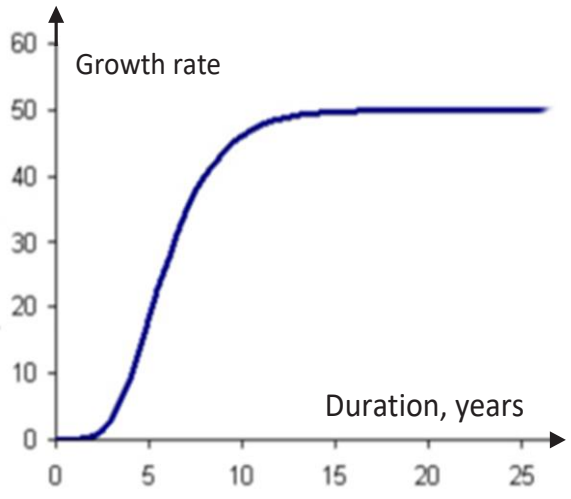

Fig. 1. Gompertz curve

Based on the Gompertz curves, in 1965 Edwin Harrington [9] Proposed a mathematical model to describe "human desire". After 1965, the Harrington desirability curve (Fig. 3) began to be widely used in problem-solving for multicriteria decision-making.

The calculation of the value of the Harrington desirability function $F_d^{Harr}(x)$ at a given value of a given target indicator $\mathrm{y}(\mathbf{x})$ is done by the formula.

$$F_d^{Harr}(y(x)) = e^{-(e^{-(b_0+b_1 y(x))})} \quad (1)$$

In formula (2), the coefficients b_0 and b_1 are calculated for each of the targets $\mathrm{y}(\mathbf{x})$ for the specific problem, by solving two equations of type (1), for which two desires are given for two values of each of the set of targets. parameters [1,2]. Usually, the value of $\mathrm{y}^{\mathrm{I}}(\mathbf{x})$ is chosen for which $F_d^{Harr}(y^{(I)}(x)) = 0.37$, i.e. at the limit of acceptability and a second value $\mathrm{y}^{(\mathrm{II})}(\mathbf{x})$ for the most desired value, for example $F_d^{Harr}(y^{(II)}(x)) = 0.98$.

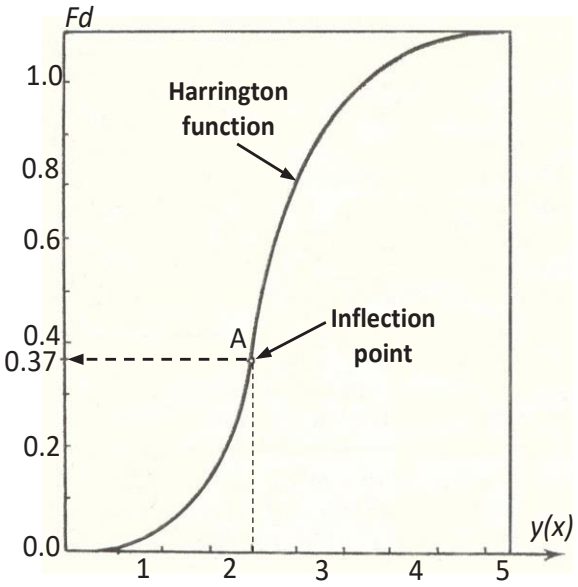

Fig. 2. Harrington function

In Fig. 2. the numerical estimates of the desired values of the objective function between the worst and the best are plotted on the ordinate axis. The boundary between "bad" and "good" rating is the inflection point "A", at which the desirability rating is 0.37. This is the point with the greatest rate of change of desire. The abscissa shows no values, only the numbers of the increasing values of the objective function $\mathrm{y}(\mathbf{x})$.

Deringer and Switch in 1980 [10] developed a series of modifications of Harrington's desirability function for different situations in solving multicriteria optimization problems in the practice. The exponential curves of the first order, which can be obtained by formula (2) are shown on the Fig. 3. On the Fig. 3 is

given also the simplest transformation in desirability of each of the target indicators $y(\mathbf{x})$ - the linear transformation, which is obtained at $r = 1$.

$$F_d^{exp}(\mathbf{x}) = \left(\frac{y^{ref}(x) - y(\mathbf{x})}{y_{max}(x) - y_{min}(x)} \right)^r, \qquad (2)$$

where $F_d^{exp}(\mathbf{x})$ is the generalized desirability function of the targets;

$y^{ref}(\mathbf{x})$ is the recommended setpoint for each target;

$y_{max}(\mathbf{x})$ и $y_{min}(\mathbf{x})$ are the allowable limits of variation (maximum and minimum) of each target.

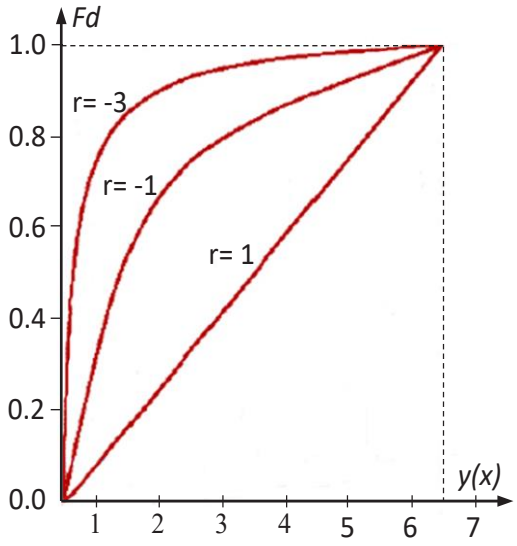

Fig. 3. Curves of Derringer and Suich.

The Harrington desirability curve [9] (Fig. 2) shows that the desired value approaches the superficial very smoothly and requires significant effort and resources for control/management to achieve it. Therefore, in the present article in the proposed new strategies for optimal solutions using a reduction of 2 % of the value of the superior, i.e. the maximum or minimum value of desirability, depending on the requirements of the respective target parameter. This value is considered the most desirable for each individual criterion $y_j(\mathbf{x})$, $j = 1, 2, \ldots, m$. The most unacceptable value is also reduced by 2 % (see Table 1).

TRANSFORMATIONS IN THE DESIRABILITY FUNCTIONS

The present study includes three types of transformations for normalization of the target indicators $y_j(\mathbf{x})$ in dimensionless values, which are used to form different types of generalized functions of desirability (or utility) $F_{dj}(\mathbf{x})$, which vary in the range (0 + 1):

(1) Linear transformation (Fig. 4);
(2) Exponential transformation (Fig. 5);
(3) Double-exponential Harrington transformation (Fig. 6).

The aim of the present study is to investigate different types of transformations of the set of target indicators $y_j(\mathbf{x})$, $j = 1,2, \ldots, m$ into generalized desirability function and to make a comparative analysis of the sensitivity of the generalized functions to changes in control parameters x_i, $i = 1,2, \ldots, n$.

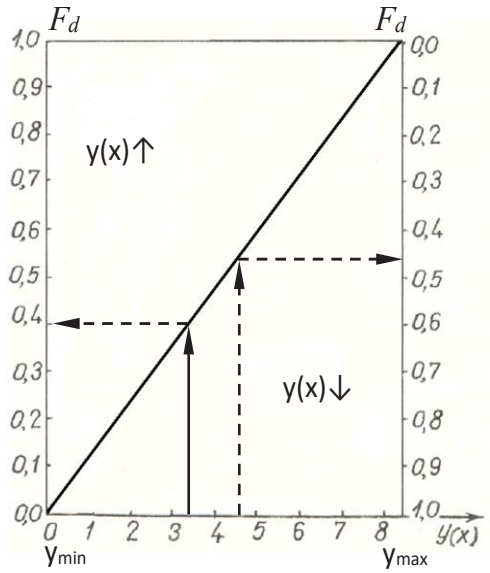

Fig. 4. A linear transformation of y(**x**) in the degree of desirability F_d

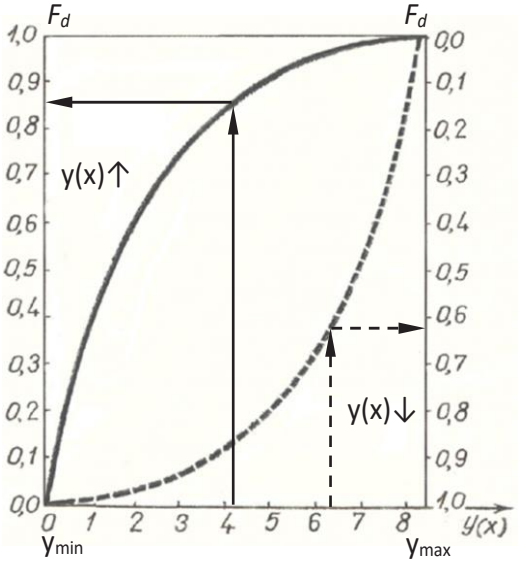

Fig. 5. Exponential transformation of $y(\mathbf{x})$ in the degree of desirability F_d.

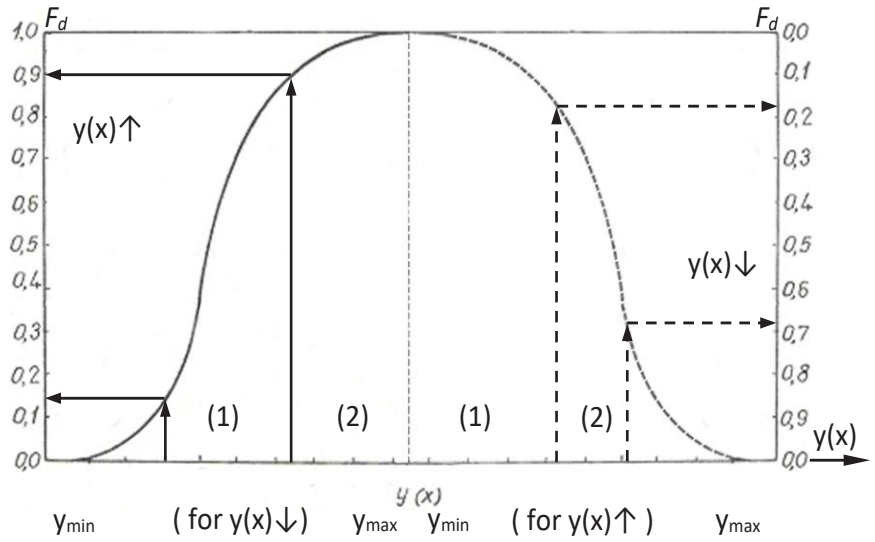

Fig. 6. Transformation of $y(x)$ in a degree of desirability F_d by a function of Harrington

GENERALIZED FUNCTIONS OF DESIRABILITY

The following kind of generalized functions of desirability are proposed and investigated for $y_j(x)$, $j = 1,2, \ldots, m$:

(a) Arithmetical mean generalized function of desirability

$$F\left(y(x)\right)_{sr.ar.} = \sum_{j=1}^{m} y_j(x) / m \qquad (3)$$

(b) Multiplicative generalized objective function

$$F\left(y(x)\right)_{mult} = \prod_{j=1}^{m} y_j(x) / m \qquad (4)$$

(c) Geometric mean generalized objective function

$$F\left(y(x)\right)_{gm} = \sqrt[m]{\prod_{j=1}^{m} y_j(x)} \qquad (5)$$

(d) Harmonic generalized objective function

$$F(y(x))_h = \frac{1}{\sum_{j=1}^{m} \frac{1}{y_j(x)}} / m \qquad (6)$$

(e) Harmonic mean generalized objective function

$$F(y(x))_{gh} = \frac{m}{\sum_{j=1}^{m} \frac{1}{y_j(x)}} \qquad (7)$$

In the examples given below the generalized function of desirability $F(y(x))$ are given in shortened form as F_d.

EXAMPLE FOR APPLICATION OF THE LINEAR, EXPONENTIAL AND DOUBLE EXPONENTIAL GENERALIZED FUNCTIONS

As an illustrative example for application of linear, exponential and double exponential generalized functions are used real data for selecting an optimal offer for buying graphite electrodes for electrical furnaces in steel production factory. Five offers are obtained from 5 companies ($x_i = 1,2,\ldots5$), named A, B, C, D and E, which produce and sell electrodes. As a quality indicators (objective parameters) for the electrodes are accepted the follows:

* Strength of pressure , N/m^2, (y_1);
* Electrical resistance, Ω , (y_2);
* Porosity, %. (y_3);
* Module of elasticity, N/m^2, (y_4).

As an additional objective parameter is selected also the price of electrodes in \$/t (y_5).

The data for the objective parameters, the maximal $y_{j\max}(x)$ and the minimal $y_{j\min}(x)$ values of the indicators are given in Table 1. In the table are given the required directions for each parameter (maximum

or minimum) and they are marked with arrows - for maximum ↑ and for minimum ↓ ($x \in [A, B, C, D, E]$).

The natural values of the objective parameters $y_j(x)$ are transformed with a linear transformation in non dimensional values $y_j^{tr}(x)$ and taken as absolute values by formula (8) and given in the Table 1.

$$y_j^{tr}(\mathbf{x}) = \left| \frac{y_j^{ref}(x) - y_j(\mathbf{x})}{y_{j,max}(\mathbf{x}) - y_{j,min}(\mathbf{x})} \right|, \; j = 1,2,\ldots,5. \tag{8}$$

The most useless values $y_j^{ref}(x)$ for the objective parameters, depending to the desired values of each parameter (maximum or minimum) are given also in Table 1.

Linear transformation of objective functions in degree of desirability

(a) Arithmetical mean desirability function

The generalized arithmetical mean desirability function for each offer for the three transformations are calculated by formulas (7) and (8) and given in Table 2 and graphically presented on Fig. 3 and Fig. 4, 5 and 6.

(i) The price is not included

$$F_{ar.m.}^{lin} = \sum_{j=1}^{4} y_j^{lin}(x) / 4 \,, \; j = 1,2,\ldots,4 \tag{9}$$

(ii) With the price

$$F_{ar.m.p}^{lin} = \sum_{j=1}^{5} y_j^{lin}(x) / 5 \,, \; j = 1,2,\ldots,5 \tag{10}$$

(b) Exponential transformation

The transformed exponential values of the objective functions are given in Table 3 and on Fig. 8.

(c) Harrington transformation

The transformed values of the objective functions by Harrincton desirability function are given in Table 4 and on Fig. 9.

The comparative analysis of the three transformation is sowing that:

(1) The highest ranks from 1 to 3 of the fifth offers keeping the ranks for all three transformations;

(2) The arithmetical mean exponential transformation is giving higher values for the generalized objective function comparing with the linear transformation and Harrington transformation;

(3) The transformation of Harrington delimits strongly the offers classified in the lowest ranks

Table 1. The objective functions values for the five electrode offers transformed in desirability.

Offer	$y_1 \uparrow$, N/m²	$y_2 \downarrow$, Ω	$y_3 \downarrow$, %	$y_4 \uparrow$, N/m²	$y_5 \downarrow$, $/t	$y_1^{lin} \uparrow$	$y_2^{lin} \downarrow$	$y_3^{lin} \downarrow$	$y_4^{lin} \uparrow$	$y_5^{lin} \downarrow$
	.10⁶ *	.10⁻⁶ *		.10⁷ *		0.98	0.98	0.73	0.42	0.25
A	**23.3**	**6.15**	22.0	91.67	2400					
B	**14.2**	7.73	**27.4**	**79.33**	**2600**	0.02	0.70	0.02	0.02	0.02
C	15.6	8.23	24.1	**108.67**	**1800**	0.15	0.61	0.45	0.98	0.98
D	14.4	8.36	26.4	92.86	2100	0.02	0.41	0.14	0.46	0.63
F	14.7	**11.50**	**20.0**	99.63	2000	0.05	0.02	0.98	0.69	0.75
y_{jmin}	**14.2**	**6.15**	**27.4**	79.33	1800					
y_{jmax}	**23.3**	**11.5**	**20.0**	108.67	2600					
Δ_j	**9.1**	**5.35**	**7.4**	29.34	800					
$y_j^{ref}(x)$	**14.2**	**11.50**	**27.4**	79.33	2600					

$\Delta_j = y_{j,max} - y_{j,min}$, $j = 1,2,\ldots,5$

* Refer for entire column

Table 2. Generalized arithmetical mean of desirability function for linear, exponential and Harrington transformation of each offer.

	Linear transformation				Exponential transformation				Transformation of Harrington			
Offer	$F^{lin}_{ar.m.}$	**Rank**	$F^{lin}_{sr.ar.p.}$	**Rank**	$F^{exp}_{ar.m.}$	**Rank**	$F^{експ}_{ar.m.p.}$	**Rank**	$F^{Harr}_{ar.m.}$	**Rank**	$F^{Harr}_{ar.m.p.}$	**Rank**
A	**0.78**	1	**0.67**	1	**0.93**	1	0.88	1	**0.88**	1	**0.72**	1
B	**0.19**	**5**	**0.16**	**5**	0.19	**5**	**0.16**	**5**	**0.24**	**5**	**0.19**	**5**
C	**0.55**	2	**0.63**	**2**	**0.71**	2	**0.76**	**2**	**0.60**	2	**0.68**	**2**
D	**0.26**	4	**0.33**	4	0.54	4	0.62	4	**0.38**	4	**0.47**	4
F	**0.44**	**3**	**0.50**	3	**0.56**	**3**	0.64	3	**0.44**	**3**	**0.54**	3

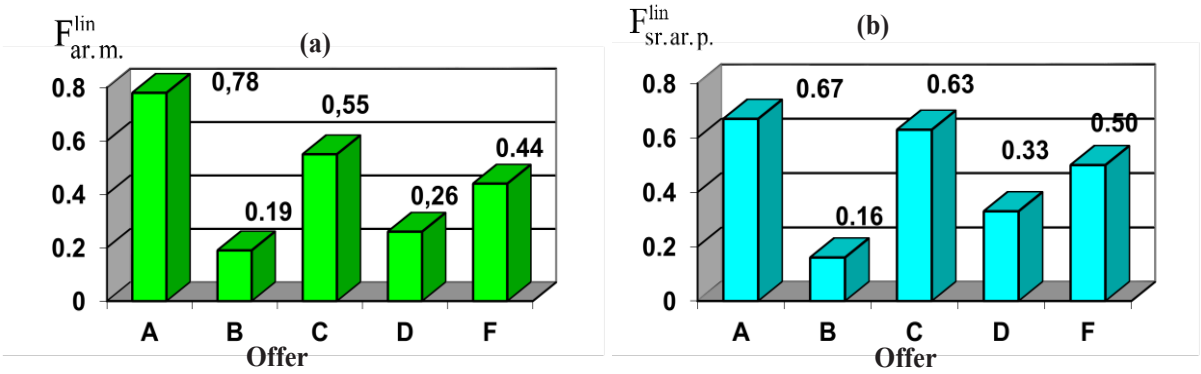

Fig. 7. Arithmetical mean linear transformation: (a) with price, (b) with out price.

Table. 3. Transformed exponential values of the objective functions.

Offer	$y_1^{exp}\uparrow$	$y_2^{exp}\downarrow$	$y_3^{exp}\downarrow$	$y_4^{exp}\uparrow$	$y_5^{exp}\downarrow$	$F^{exp}_{ar.m.}$	Rank	$F^{exp}_{ar.m.p.}$	Rank
A	**0.98**	**0.98**	**0.96**	**0.81**	**0.66**	**0.93**	1	0.88	1
B	**0.02**	**0.70**	**0.02**	**0.03**	**0.02**	0.19	**5**	**0.16**	**5**
C	0.49	**0.76**	**0.60**	**0.97**	**0.98**	**0.71**	2	**0.76**	**2**
D	0.19	**0.90**	**0,21**	**0.85**	**0.94**	0.54	4	0.62	4
F	0.29	**0,02**	**0.98**	**0.96**	**0.95**	**0.56**	**3**	0.64	3

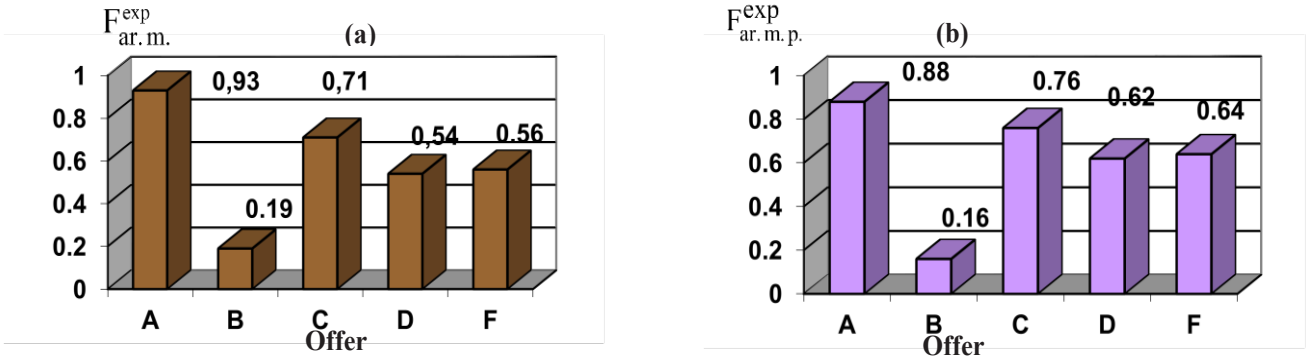

Fig. 8. Arithmetical mean exponential transformation: (a) with price, (b) with out price

Table. 4. Transformed values of the objective functions by Harrincton desirability function.

Offer	$y_1^{Harr}\uparrow$	$y_2^{Harr}\downarrow$	$y_3^{Harr}\downarrow$	$y_4^{Harr}\uparrow$	$y_5^{Harr}\downarrow$	$F^{Harr}_{sr.ar.}$	Rank	$F^{Harr}_{sr.ar.u}$	Rank
A	**0.98**	**0.98**	0.92	0.62	0.08	**0.88**	1	**0.72**	1
B	**0.02**	0.88	**0.02**	**0.02**	**0.02**	**0.24**	**5**	**0.19**	**5**
C	0.06	0.81	0.55	**0.98**	**0.98**	**0.60**	2	**0.68**	**2**
D	0.03	0.77	0.05	0.65	0.83	**0.38**	4	**0.47**	4
F	0.03	**0.02**	**0.98**	0.73	0.93	**0.44**	**3**	**0.54**	3

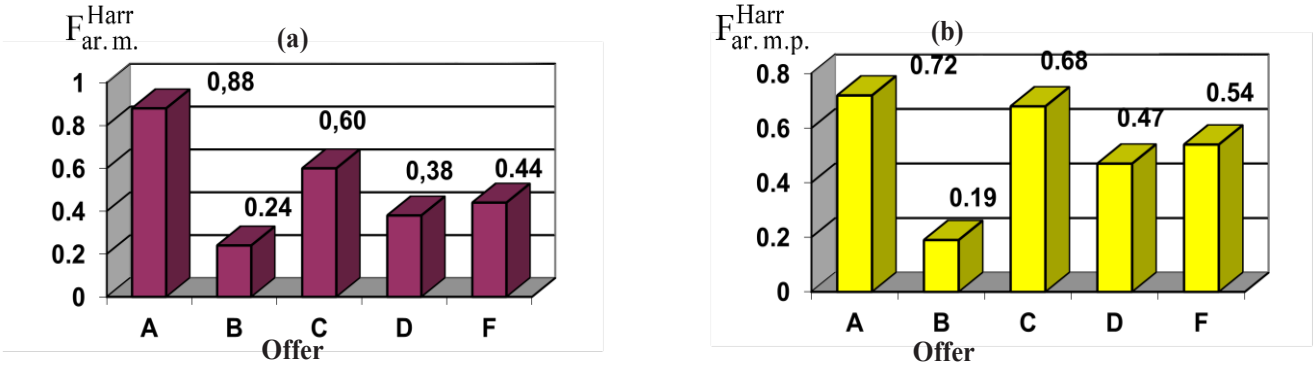

Fig. 9. Arithmetical mean Harrington transformation: (a) with price, (b) with out price.

CONCLUSION

(1) Five different kinds of creation of generalized objective functions for several criteria decision making are proposed;

(2) Three strategies for transformation of several objective parameters in one generalized desirability function are proposed:

* Linear transformation;

* Exponential transformation;

* Function of desirability of Harrington.

(3) Three strategies for optimal decision making are investigated on the real practical example of choosing optimal offer among five proposals. The results are showing:

(a) The highest first tree ranks are equal for the first three offers;

(b) The arithmetical mean exponential transformation is giving higher values comparing with linear transformation and transformation of Harrington;

(c) The Harrington transformation differs strongly the offers classified on lowest ranks;

(d) Including the price in the generalized objective desirability function keeps the ranks of the offers, which is showing that the trade prices of the offers correspond to the quality of the electrodes.

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