Priorities and weight coefficients determining for the quality indicators of quartz ramming masses

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Abstract. Taking into account the priorities of the objective indicators in the compromise decision-making for multi-criteria optimization tasks makes it possible to reduce the number of indicators. In the present study, an algorithm for determining the weight coefficients of the objective indicators based on the opinions of experts and the possibility to reduce the number of indicators by eliminating those with the lowest priority is proposed. It is proposed to do this in combination with the study of the correlation between the objective parameters

Keywords: compromise solutions, enquiry card, linear correlation, rank correlation, matrix of ranks, Kendall's coefficient, coefficient of consent, weight coefficients, priorities estimation, ramming masses for steel foundry ladles

I. Introduction

In order to find compromise solutions for the optimal technological process in the production of quartz ramming masses, it is necessary to assess the priorities of the quality indicators of the ramming masses [1, 2, 3, 4]. This can be done by studying the opinion of experts and using rank correlation methods [6, 8]. It is necessary to consult a survey of specialists on their subjective opinion on the priorities of quality indicators of ramming mases. The opinions from the specialists should be studied for the presence of the opinions concordance. If there is significant consent in the opinions, then an objective assessment of the priorities of the indicators can be made and weight coefficient for the quality indicators of the ramming masses can be calculated.

II. Quality indicators survey

To determine the priorities and weight coefficients of the quality indicators of quartz ramming masses, a number of enquired experts, including R = 10 experts, who were asked to rank in order of importance of m = 8 criteria for the quality of ramming masses for steel foundry ladles was done. Each expert was asked to give an anonymous opinion on the ranking of each of the criteria. The ten experts were asked to fill in a questionnaire in which to fills his opinion on the ranks of each indicator with numbers from 1 to 8 (Table 1). The highest rank (priority) is **1**.

TABLE I. Enquiry card for quality indicators priority of the ceramic ramming masses for steel foundry ladles determination

Indicator	Objective parameter	Rank
Linear changes, %	Y_1	
Open porosity, %	Y_2	
Apparent density, g/cm^3	Y_3	
Compressive strength, MPa	Y_4	
Beginning of deformation, °C	Y_5	
4% Deformation, °C	Y_6	
40% Deformation, °C	Y_7	
Water cycles, number	Y_8	

The ranking matrix of opinions is created from the received opinions. The results obtained from the survey are given in Table. 2.

III. Evaluation of the coefficient of consent (concordance)

To determine the coefficient of consent (coefficient of concordance), a matrix of ranks was created (Table 2). In each row of the table, the experts gave their opinion regarding the priorities of the indicators from Y_1 to Y_8.

TABLE II. MATRIX OF RANKS OF THE QUALITY INDICATORS OF CERAMIC RAMMING MASSES

Quality indicator → Expert ↓	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8
1	3	2	7	5	1	4	6	8
2	2	3	8	6	1	4	5	7
3	1	3	7	6	2	5	4	8
4	5	3	8	6	4	2	1	7
5	2	1	6	7	3	4	5	8
6	3	2	7	6	1	5	4	8
7	4	2	8	6	1	3	5	7
8	3	1	7	5	2	4	6	8
9	4	1	6	7	5	3	2	8
10	3	4	6	7	1	2	5	8
$\sum_{i=1}^{10} \alpha_{ij}$	30	22	70	61	21	36	43	77
Δ_j	-15	-23	25	16	-24	-9	-2	32
V_j	0,714	0,826	0,143	0,271	0,843	0,628	0,527	0,043
W_j	**0,179**	**0,207**	**0,036**	**0,068**	**0.211**	**0,157**	**0,132**	**0,010**
Rank	**3**	**2**	**7**	**6**	**1**	**4**	**5**	**8**

Algorithm for coefficient of consent evaluation

(1) Rank matrix creation (Table. 2)

(2) Average sum of all ranks is calculated

$$S_{aver} = \frac{R(m+1)}{2} = \frac{10(8+1)}{2} = 45 \quad (1)$$

(3) The sum $\sum_{i=1}^{10} \alpha_{ij}$ and the deviations

$\Delta_j = \sum_{i=1}^{10} \alpha_{ij} - S_{aver}$ are calculated.

The results are given in Table.2;

(4) The sum $\sum_{j=1}^{8} \Delta_j^2 = 3320$ is calculated.

(5) The Kendall coefficient of consent is calculated [6].

$$w_k = \frac{12\sum_{j=1}^{m} \Delta_j^2}{R^2(m^3 - m)} = \frac{12.3320}{10^2(8^3-8)} = 0,79 \quad (2)$$

(6) Determining the significance of the coefficient of consent.

The statistical significance of the coefficient of consent (coefficient of concordance) w_k is proved by Z-criterion [7], if the number of indicators is m ≤ 7, or by χ^2-criterion, if the number of indicators m > 7.

Since in the example the number of the quality indicator m is greater than 7 (m = 8). The calculated value of is χ^2_{calc} is:

$$\chi^2_{calc} = 10.(8-1).0,79 = 55,3 \quad (3)$$

With an accepted level of significance α = 0,01 (confidence probability β = 0,99) and number of degrees of freedom v = 8 – 1 = 7, the tabular value of χ^2 is $\chi^2_{calc} = 18,475$.

$$\chi^2_{calc} = 55,3 > 18,475 = \chi^2_{tabl} \quad (4)$$

The coefficient of consent w_k is considered significant because $\chi^2_{calc} > \chi^2_{tabl}$, with a probability not least than 99% there is concordance in the opinions of the surveyed specialists and the weight coefficients of the quality indicators can be assessed objectively.

IV. Calculation of Weight Coeficients for the Quality Indicators of the Ramming Masses

As there is concordance in the subjective opinions of the surveyed specialists, the weight coefficients can be calculated according to the formulas [8]:

$$V_j = \frac{R.m - \sum_{i=1}^{R} \alpha_{ij}}{R.m - m} \quad (5)$$

$$W_j = \frac{V_j}{\sum_{j=1}^{m} V_j}, \; j = 1, 2, \ldots, m, \quad (6)$$

The condition for the weight coefficients calculated according to formula (6) exists the following condition:

$$\sum_{j=1}^{m} W_j = 1 \quad (7)$$

The calculated weight coefficients for the example are given in Table. 2 and on a Fig. 1.

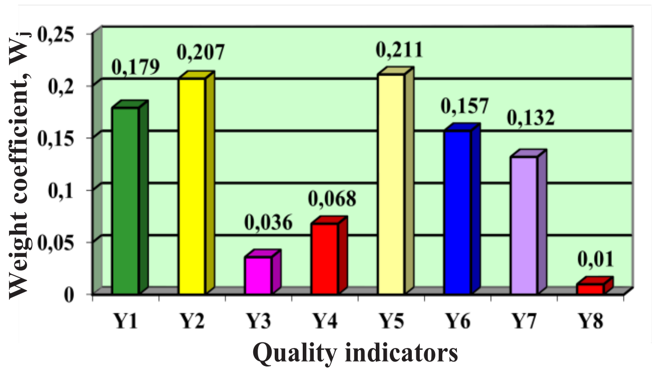

Fig. 1. Weight coefficients of the quality indicators of the ramming masses type "Fosuk"

The ranking of the quality indicators of the ceramic ramming masses are given in Table. 3.

The most important are the criteria: beginning of deformation - Y_5, with a weight coefficient W_5=0,211; open porosity - Y_2, with a weight coefficient W_2=0,207 and the criterion linear changes of the ramming masses - Y_1, with a weight coefficient W_1=0,179.

The objective indicators that can be eliminated are Y_8 (Water cycles) and Y_3 (Apparent density).

TABLE III. Ranking of the priorities of the quality indicators of the quartz ramming masses

-	Quality indicator	Dimension	Rank
Y_5	Beginning of deformation	°C	(1)
Y_2	Open porosity	%	(2)
Y_1	Linear changes	%	(3)
Y_6	4% Deformation	°C	(4)
Y_7	40% Deformation	°C	(5)
Y_4	Compressive strength	MPa	(6)
Y_3	Apparent density	g/cm^3	(7)
Y_8	Water cycles	number	(8)

The ranks from Table. 3 and the estimated weight coefficients from Table. 2 will be used for the Pareto-optimal (compromise) solutions determining [5,8] for the control parameters: clay content (%Al_2O_3) and the heat treatment temperature of the masses °C.

V. Linear Correlation of the Quality Indicators of the Ramming Masses

Optimal decision making is more difficult when the number of objective parameters is greater. Their number can be reduced in two ways:

(a) By eliminating the objective parameters with low-priority, ie low weight coefficients;

(b) By studying the linear correlation between every two objective parameters and eliminating some of them. For example, if two of the objective parameters Y_2 and Y_5 are linearly correlated with a positive sign of a statistically significant correlation coefficient $R_{2\text{-}5}$ and the requirements for the both parameters regarding desired values are to be highest it means that when the control parameters X_1 and X_2 are changed to increase Y_2 simultaneously Y_5 will be increased also.

From Table. 4 it can be seen that many of the objective indicators are strongly linear correlated and this may give sufficient prove to the experts to reduce the number of indicators in determining the optimal process control of manufacturing quartz ramming masses intended for steel foundry ladles.

TABLE IV. COEFFICIENTS OF LINEAR CORRELATION R_{I-J} BETWEEN THE QUALITY INDICATORS OF QUARTZ RAMMING MASSES FOR STEEL FOUNDRY LADLES

-	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8
Y_1		-	**-0,698**	+0,450	**+0,665**	**+0,814**	**+0,814**	-0,464
Y_2	-		-0,614	**-0,858**	-	-	-	-
Y_3	**-0,698**	-0,614		-	-	-	-0,449	-
Y_4	+0,450	**-0,858**	-		+0,579	+0,574	+0,574	-
Y_5	**+0,665**	-	-	+0,579		**+0,986**	**+0,951**	**-0,751**
Y_6	**+0,814**	-	-	+0,574	**+0,986**		**+0,981**	**-0,702**
Y_7	**+0,814**	-	-0,449	+0,574	**+0,951**	**+0,981**		**-0,703**
Y_8	-0,464	-	-	-	**-0,751**	**-0,702**	**-0,703**	

* For the values, marked in blue, the confidence level is 95%

** For the values marked in red, the confidence level is 99%

VI. CONCLUSION

Optimal decision-making in the presence of many objective parameters is an incorrect task because it does not have a single solution. Difficulties in choosing the optimal solution increase with the number of objective parameters and the subjective contradictions that arise between stakeholders in making a particular decision. The present study on a real technological problem proposes using of subjective opinions and the objective assessment of their concordance to rank the priorities of the objective indicators when making compromise decisions.

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